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HEAT TRANSFER MEASUREMENTS IN CYLINDRICAL WALL JETS

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Abstract—In this article measurements of mean velocities and heat-transfer rates in a cylindrical wall jet formed by a jet emerging through a convergent nozzle are presented. The heat-transfer rate and the maximum velocity decay are influenced by the transverse curvature present. The experimental results are correlated successfully by using the diameter of the cylinder to slot height ratio as the curvature parameter. With a diameter to slot height ratio of 0·125, the Stanton number values for a cylindrical wall jet are as much as 1·7 times that of a plane wall jet, under similar conditions.

NOMENCLATURE

a,	radius of the cylinder;
C_p,	specific heat at constant pressure;
h,	local heat-transfer coefficient;
$N_{Re, a}$,	local radius Reynolds number, $U_m a/\nu$;
$N_{Re, m}$,	inner layer Reynolds number, $U_m \delta_m/\nu$;
$N_{Re, S}$,	slot Reynolds number, $U_s S/\nu$;
$N_{St, S}$,	Stanton number, $h/\rho C_p U_s$;
$N_{St, m}$,	Stanton number $h/\rho C_p U_m$;
S,	annular slot height;
U,	velocity;
U_S,	velocity at the nozzle exit;
U_m,	local maximum velocity;
U_p,	velocity in the Plenum chamber;
X,	distance downstream measured from the nozzle exit;
X_0,	distance between the virtual origin and nozzle exit;
Y,	radial distance from the wall;
$\alpha\beta$,	constants defined in equation (2);
δ,	radial distance from the surface of the cylinder to the point in the outer region at which $U = U_m/2$;
δ_m,	radial distance from the surface of the cylinder to the point in the outer region at which $U = U_m$;
ν,	kinematic viscosity;
ξ,	distance downstream along the cylinder from the imaginary origin, $\xi = x + x_0$;
ρ,	density;
τ_ω,	wall shear stress;
$\Delta k_1, \Delta k_2$,	constants defined in equation (5) and (6).

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INTRODUCTION

A CYLINDRICAL wall jet is created when a jet emerges from an annular slot and flows longitudinally and coaxially along a cylinder. This paper presents the results of an experimental investigation for the velocity field and the heat transfer characteristics of such a fully developed turbulent cylindrical wall jet.

A number of investigators have carried out experimental and theoretical investigations on cylindrical wall jets. McDonald [1] has reported an analysis for the laminar wall jets on slender bodies of revolution and experimental results for the skin friction and heat-transfer coefficients on a cylindrical wall jet flow characteristics. Singh [2] and Akatnov [3] independently carried out integral analyses for the turbulent cylindrical wall jet flow characteristics. Both of them assumed that the inner boundary layer in a cylindrical wall jet behaves like a boundary layer over a flat plate. Experiments were also carried out by Singh [2] and Akatnov [3] using 1 in., $3\frac{1}{2}$ in. and 10 in. and 6 cm and 10 cm diameter cylinders respectively. All the above mentioned experiments do not show any increase in the local heat transfer and skin friction coefficients over that of a plane wall jet. Experimental results using a $\frac{1}{2}$ in. diameter cylinder, reported by Starr and Sparrow [4], show increases as much as 180 per cent in the local skin friction coefficients for the cylindrical wall jet over that for a plane wall jet at the same inner layer Reynolds number ($U_m\delta_m/\nu$). There appears to be no published work available establishing the effect of transverse curvature on the heat-transfer characteristics of wall jets.

This work establishes the effect of the transverse curvature on the heat-transfer characteristics of wall jets and extends the available data on the velocity field.

EXPERIMENTS

The experiments were carried out on a horizontal cylindrical wall jet issuing through a convergent annular nozzle as illustrated in Fig. 1. A blower with a variable speed drive

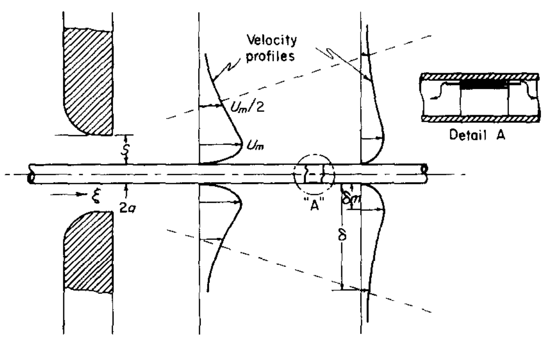

FIG. 1. Schematic diagram of the circular wall jet.

and a plenum chamber provided the necessary air supply at a temperature controlled to within $\pm\frac{1}{4}$°F of the room temperature. The plenum chamber which contained three screens in series provided a uniform flow at the nozzle exit. All the nozzles used were as per ASME code PTC19.5; 4-1959 and were machined out of plastic. Turbulence intensity and flow uniformity were checked at the nozzle exit by measurement. The measurements revealed a maximum turbulence intensity of 0·8 per cent.

Stainless steel thin walled tubing of $\frac{1}{8}$ in. and $\frac{1}{4}$ in. o.d. were used as test cylinders. These tubes were put under tension from a ball and socket type support at the ends. This arrangement permitted a fine alignment without altering the tension. The tubes were heated by passing a current through them from a regulated a.c. power supply. This permitted an accurate determination of the heat flux at the wall.

A single copper–constantan thermocouple (0·013 in. wire diameter) embedded in a circular plastic insert was introduced inside the thin walled tubing, as shown in Fig. 1, in order to measure inside wall temperature at different axial locations. The necessary correction to the thermocouple readings to obtain the outside wall temperature was negligible when calculated using the analysis given in [5]. It was estimated that the wall temperature measurements are correct to within $\pm 0{\cdot}3$°F. Velocity measurements were taken using a circular total pressure tube (0·012 in. i.d. and 0·022 in. o.d.) and the traversing mechanism described in [1].

In all, twenty heat-transfer tests and ten velocity profile survey tests were made. Table 1 gives the complete details of all the tests. For each test, cylinder alignment was verified by measuring the velocity profiles in three different circumferential locations in the middle of the test section. Boundary layer parameters (δ, δ_m, U_m) were calculated using the calculation procedure mentioned in [4]. A more detailed description of the apparatus, test procedure and data reduction will be found in [6].

VELOCITY PROFILE RESULTS

Representative, non-dimensional, fully de-

Table 1

$2a$	$\frac{1}{8}$ in.				$\frac{1}{8}$ in.				$\frac{1}{8}$ in.				$\frac{1}{8}$ in.				$\frac{1}{4}$ in.			
$2a/s$	2				1				0·285				0·125				0·666			
u_p/u_s	0·0014				0·00039				0·003				0·014				0·003			
Velocity field test number		1		2		3		4		5		6		7		8		9		10
$N_{Re,S}^{10^{-3}}$	1·5	2·75	3·9	5·1	3·7	5·3	7·8	10·6	13·8	19·3	27·3	36·9	29·5	43·6	62·2	83·1	11·5	16·4	23	32·7
Nomenclature	○	□	◇	△	◺	D	∩	◇	◊	⌂	∆	⬡	0	▽	◸	D	U	⬠	▽	⬭
Heat transfer test number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
ξ/s Range	32–380				32–200				22–125				24–56				26–144			
	3·2				5·6				2·05				3				2·67			

veloped, velocity profile results are shown in Fig. 2. It is seen that the velocity profiles are similar in the outer mixing layer. The profiles when plotted to a larger scale were not similar in the inner boundary layer. The overall non-similarity of the velocity profile is shown

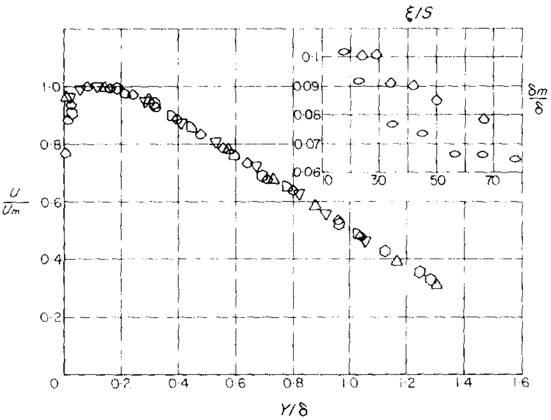

FIG. 2. Non-dimensional mean velocity profiles (for nomenclature see Table 1).

through a plot of the ratio δ_m/δ against the non-dimensional length ξ/S, (insert, Fig. 2). Seban and Back [7] used a value of 0·120 for this ratio along the entire length of a plane wall jet. However, for cylindrical wall jets, this ratio is not constant due to the non-similarity of the velocity profiles. This has already been observed by McDonald [1] and Starr and Sparrow [4], on larger diameter cylinders.

The streamwise development of the flow field is characteristized by the jet spread parameter, δ, and maximum velocity decay index, U_m/U_s. One limiting case, $(a \to \infty)$, of a cylindrical wall jet is the plane wall jet. In the other limiting case, $(a \to 0)$, the cylindrical wall jet tends towards a free circular jet in the outer layer. Experiments on the plane wall jet and the free circular jet indicate that:

for the free circular jet $U_m \propto \xi^{-1}$, $\delta \propto \xi$
and for the plane wall jet $U_m \propto \xi^{-0.5}$, $\delta \propto \xi$

where ξ is the distance downstream measured from the virtual origin of the jet.

It is seen from Fig. 3 that the jet spread obeys a simple power law relation.

$$\xi/S = 0{\cdot}125\,(\xi/S)^{0{\cdot}900}, \tag{1}$$

as in the case of the free circular jet and the plane wall jet.† However, the maximum velocity

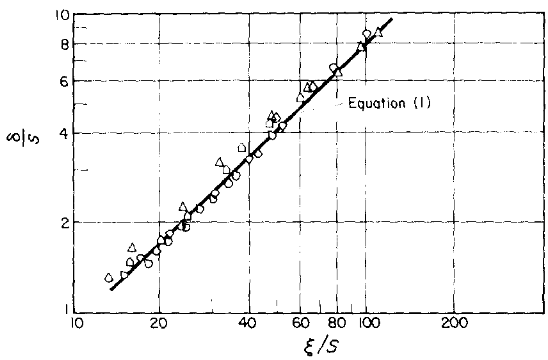

FIG. 3. Jet spread (for nomenclature see Table 1).

decay does not obey a simple law relation, as it is seen to be influenced by the ratio $2a/S$. A cylindrical wall jet can be expected to behave like a plane wall jet when the ratio δ/a is large. This is well illustrated by the maximum velocity decay characteristics. It is seen with the smallest $2a/S$ ratio, or for large value of δ/a, that the maximum velocity decay is almost like a free circular jet. With increasing $2a/S$ or decreasing δ/a, the results tend towards those of a plane wall jet. The experimental results of McDonald [1] and Starr and Sparrow [4] plotted in Fig. 4 also confirm this result. It must be noted that the slot height has no influence on the maximum velocity decay in the case of a plane wall jet.

HEAT-TRANSFER RESULTS

Heat-transfer data on wall jets are usually presented in the form

$$N_{St,S} = K(N_{Re,S})^{-\alpha}(\xi/S)^{-\beta}. \tag{2}$$

† The distance between the virtual origin and the nozzle exit was determined by a linear extrapolation of the curve of δ vs. x to zero thickness.

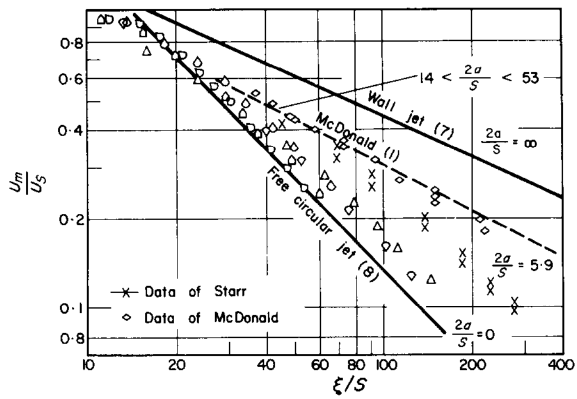

FIG. 4. Decay of the maximum velocity.

The present experiments used an unheated jet and a constant heat flux wall. Saban and Back [7], based on their experiments on an unheated plane wall jet with a constant heat flux wall, recommend $K = 0{\cdot}41$, $\alpha = 0{\cdot}3$ and $\beta = 0{\cdot}6$. Dimensional analysis shows that the heat transfer is also influenced by the ratio $2a/S$. Figure 5 shows the data together with the least

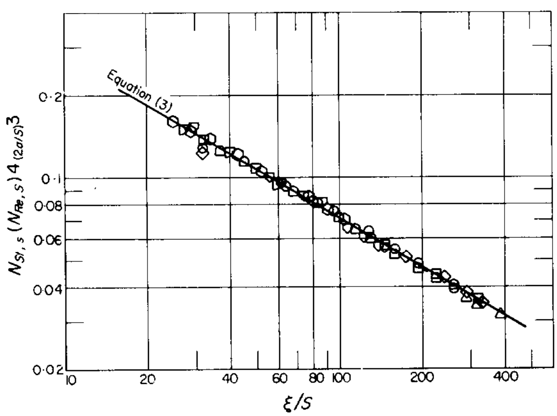

FIG. 5. Heat-transfer results as per equation (3) (for nomenclature see Table 1).

square line through it. The equation for this least square straight line is

$$N_{St,S} = 1{\cdot}27\,(N_{Re,S})^{-0{\cdot}4}\,(\xi/S)^{-0{\cdot}6}\,(2a/S)^{0{\cdot}3} \qquad (3)$$

Ninety-five per cent of the data points fall within ± 5 per cent of this line, for $25 < \xi/S < 380$ and $0{\cdot}03 < N_{St,S}N_{Re,S}^{0{\cdot}4}\,(2a/S)^{0{\cdot}3} < 0{\cdot}15$.

It was previously mentioned that the ratio δ_m/δ is smaller in a cylindrical wall jet than in a plane wall jet. This implies that the cylindrical wall jet heat-transfer rates must be greater than the plane wall jet heat-transfer rates under similar conditions. A comparison can be made in two different ways, as given below.

The heat-transfer results for plane wall jets of Seban and Back [7] can also be represented in the form,

$$N_{St,m} = 0{\cdot}047\,N_{Re,m}^{-0{\cdot}25}. \qquad (4)$$

For comparison with plane wall jets, the present data are correlated in the form,

$$N_{St,m} = (0{\cdot}047 + \Delta k_1)\,N_{Re,m}^{-0{\cdot}25} \qquad (5)$$

and Δk_1 is calculated for test conditions for which both fluid flow and heat-transfer measurements are made. The values of Δk_1 are shown in Fig. 6. It may be noted that the skin friction results

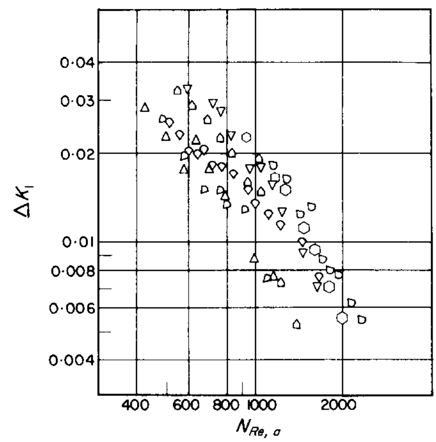

FIG. 6. Heat-transfer results comparison with the plane wall jet (for nomenclature see Table 1).

of Starr and Sparrow [4] were correlated successfully by

$$\frac{2\tau_\omega}{\rho U_m^2} = \left\{0{\cdot}0565 + 48{\cdot}1 \times \left[a\frac{\sqrt{(\tau_w/\rho)}}{\nu}\right]^{-1{\cdot}27}\right\} N_{Re,m}^{-0{\cdot}25}$$

Their results were obtained with a slot height factor of 5·9. On the current heat-transfer results the influence of slot height factor is seen to be that as $2a/S$ decreases Δk_1 increases for a given $N_{Re,a}$ and $N_{Re,m}$. Hence it would appear that the correlation of [4] would be valid only for a slot height factor of 5·9, if an analogy exists. Based on equation (2) the second method of comparison is the calculation of Δk_2 in the correlation,

$$N_{St,S} = (0{\cdot}41 + \Delta k_2) N_{Re,S}^{-0{\cdot}3} (\xi/S)^{0{\cdot}6}. \quad (6)$$

Values of Δk_2 as a function of the ratio $2a/S$ were calculated from the experimental results and an expression for Δk_2 was obtained. Replacing Δk_2 as a function of $2a/S$ equation (6) becomes

$$N_{St,S} = (0{\cdot}41 + 0{\cdot}057\,(2a/S)^{-0{\cdot}84}) N_{Re,S}^{-0{\cdot}3} (\xi/S)^{-0{\cdot}6}. \quad (7)$$

With the minimum value of slot height factor used in the experiments, equation (7) indicates a 70 per cent increase in the heat-transfer rate over that of plane wall jet at a given $N_{Re,S}$ and ξ/S. Equation (7) is shown plotted in Fig. 7

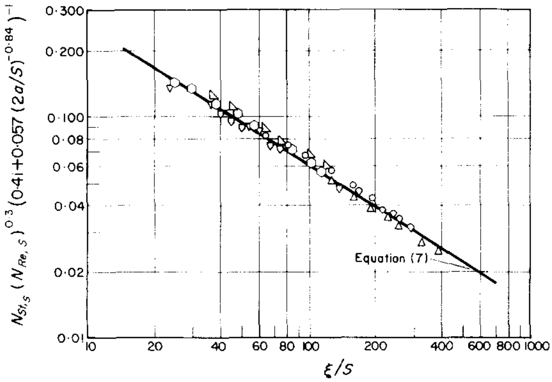

FIG. 7. Heat-transfer results as per equation (7) (for nomenclature see Table 1).

together with the data. Equations (3) and (7) appear to correlate the heat-transfer results equally well as can be seen by comparing Figs. 5 and 7. The equations agree with each other to within 5 per cent in the experimental range except when $2a/S = 2$, for which the maximum discrepancy is about 10 per cent which occurs when $N_{Re,S} = 5{\cdot}1 \times 10^3$. Also it is noted that equation (3) involves fewer empirical constants.

CONCLUSIONS

The transverse curvature present in a cylindrical wall jet has no effect on the jet spread, but it does affect the velocity profile similarly, the rate of decay of the maximum velocity and the local heat-transfer rate. In a cylindrical wall jet the velocity profiles are non-similar, as shown in the insert in Fig. 2. The maximum velocity decay does not obey a simple power law because it is influenced by the amount of curvature present, represented by diameter to slot height ratio, $2a/S$, as shown in Fig. 4. The heat-transfer results are correlated successfully with the slot height factor as the curvature parameter as in equation (3) or equation (7). The Stanton number values for a cylindrical wall jet are as much as 1·7 times that of a plane wall jet, as given by Fig. 7 and by equation (7), at the same time inner layer Reynolds number or at the same slot Reynolds number and non-dimensional length ξ/S.

REFERENCES

1. T. W. McDONALD, The submerged surface jet, Ph.D. Thesis, Purdue University, August 1965.
2. P. P. SINGH, The submerged cylindrical surface jet, Ph.D. Thesis, University of Saskatchewan, May 1966.
3. N. I. AKATNOV, Propagation of an axially symmetrical turbulent stream on the external surface of a circular cylinder along its generator, (in Russian) *Refratinsnyi Zh. Mekh.* 6, *Rev.* 6B511 (1962).
4. J. B. STARR and E. M. SPARROW, Experiments on a turbulent cylindrical wall jet, *J. Fluid Mech.* **29**, 495–512 (1967). (See also J. B. STARR, Ph.D. Thesis, University of Minnesota, 1967).
5. R. E. LUNDBERG, W. C. REYNOLDS and W. M. KAYS, Heat transfer with laminar flow in concentric annuli with constant and variable wall temperature with heat NASA TN D-1972 (1963).
6. V. S. MANIAN, Ph.D. Thesis, Cylindrical wall jet, University of Saskatchewan.
7. R. A. SEBAN and L. H. BACK, Velocity and temperature profiles in a wall jet, *Int. J. Heat Mass Transfer*, **3**, 255–265 (1961).
8. N. TRENTACOSTE and P. M. SFORZA, An experimental investigation of three dimensional free mixing in incompressible free jets, Polytechnic Institute of Brooklyn, *Pibal Rep.* No. 871 (1966).

Résumé—Dans cet article, on présente des mesures de vitesses moyennes et de densité de flux de chaleur dans un jet pariétal cylindrique formé par un jet sortant d'une tuyère convergente. La densité de flux de chaleur et la décroissance de la vitesse maximale sont influencées par la courbure transversale actuelle. Les résultats expérimentaux sont corrélés avec succès en employant le rapport du diamètre du cylindre à la hauteur de la fente comme paramètre de courbure. Avec un rapport diamètre sur hauteur de fente de 0,125, les valeurs du nombre de Stanton pour un jet pariétal cylindrique peuvent aller jusqu'à 1,7 fois celles d'un jet pariétal plan, sous des conditions semblables.

Zusammenfassung—In diesem Artikel werden Messungen von mittleren Geschwindigkeiten und Wärmeströmen in einem durch eine zylindrische Wand im Zentrum begrenzten Freistrahl behandelt, der aus einer konvergenten Ringspaltdüse kommt. Der Wärmestrom und der maximale Geschwindigkeitsabfall werden durch die vorhandene Krümmung quer zum Strom beeinflusst. Die experimentellen Ergebnisse können gut korreliert werden, wenn man das Verhältnis Durchmesser des Zylinders zu Spalthöhe als Kurvenparameter benutzt. Bei einem Verhältnis Durchmesser zu Spalthöhe von 0,125 ergeben sich unter ähnlichen Bedingungen. Werte der Stanton–Zahl für einen zylindrisch begrenzten Freistrahl, die um mehr als 70% über denjenigen für einen eben begrenzten Freistrahl liegen.

Аннотация—В данной статье приводятся результаты измерений средних скоростей и интенсивности теплообмена в струе, истекающей через сужающееся сопло вдоль цилиндрической стенки. Поперечная кривизна стенки влияет на интенсивность теплообмена и на уменьшение максимальной скорости. Экспериментальные результаты хорошо аппроксимируются с помощью отношения диаметра цилиндра к высоте щели, которое используется в качестве параметра кривизны. При отношении диаметра к высоте щели, равном 0,125, для цилиндрической струи на стенке число Стантона в 1,7 раза больше, чем число Стантона для плоской струи на стенке в аналогичных условиях.